TRINITY COLLEGE

Year 11 Mathematics Specialist Units 1, 2
Test 5 2020

Section 1 Calculator Free
Matrices

STUDENT'S NAME ____

DATE: Wednesday 19 August **TIME:** 20 minutes **MARKS**: 20

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (5 marks)

Consider the system of equations
$$x+y=3$$
$$2x+3y=8$$

(a) Write this in the form $AX=B$ where $X=\begin{bmatrix} x \\ y \end{bmatrix}$. [2]

(b) Using matrix methods, solve $AX=B$ and solve for x and y. [3]

2. (8 marks)

Given the matrices A, B, C and D shown below, where possible, evaluate each of the following. If the expression cannot be evaluated, clearly explain why this is the case.

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & -3 \\ k & -1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 4 & 0 & 3 \end{bmatrix}$$

(a) $A + 2C$ [2]

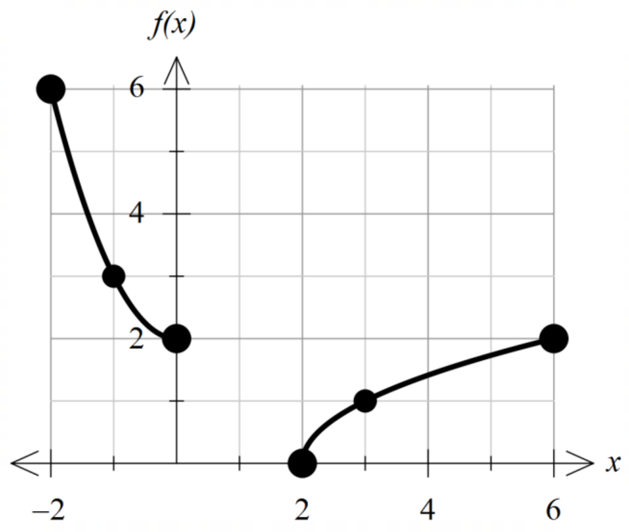
(b) DB [2]

(c) CA [2]

(d) The value of k such that CA is singular. [2]

3. (7 marks)

The points $(-2,6)$, $(-1,3)$ and $(0,2)$ on the graph of the function $f(x) = x^2 + 2$ are moved by a linear transformation as shown in the diagram below:



(a) State the appropriate transformation matrix. [1]

(b) Under a second transformation, $(2,0)$ and $(3,1)$ become $(2,0)$ and $(5,1)$ respectively.

(i) Determine the matrix that will achieve this. [2]

(ii) State the new coordinates of $(6,2)$. [1]

(c) Determine the transformation matrix which will transform the points from their final positions in part (b) back to their original positions. [3]

TRINITY COLLEGE

Year 11 Mathematics Specialist Units 1, 2
Test 5 2020

Section 2 Calculator Assumed
Matrices

STUDENT'S NAME ______________________________

DATE: Wednesday 19 March **TIME:** 30 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

4. (4 marks)

Solve the equation $X\begin{bmatrix} 2 & -2 \\ -7 & 4 \end{bmatrix} + X = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}$ for the 2×2 matrix X

5. (10 marks)

The triangle OAB is defined by the points $O(0,0)$, $A(5,0)$ and $B(4,3)$.

(a) Determine the coordinates of the points O', A', B' of the triangle when it is transformed by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. Describe this transformation geometrically. [3]

(b) The triangle $O'A'B'$ from part (a) is then transformed by a second matrix that represents a reflection about the line $y = -x$. Write the combined effect of the two transformations as a single matrix. [3]

(c) The original triangle OAB is transformed by a dilation factor 3 parallel to the x-axis and a factor k, $k > 0$, parallel to the y-axis. If the resulting image has an area of 50 square units, determine the value of k. [3]

6. (8 marks)

For any matrix M its transpose M^T is obtained by interchanging the rows and columns of M .
For example

$$\text{if} \quad M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{then} \quad M^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

A matrix M is called orthogonal if $M^{-1} = M^T$.

(a) For each of the matrices that follow, write down its transpose and its inverse and, hence, decide is the matrix is orthogonal.

(i) $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ [3]

(ii) $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ [3]

(b) In part (a) you were asked to use a method involving inspection to decide whether a square matrix is orthogonal. Justify and describe another method that could be used to determine whether a square matrix is orthogonal. This method should involve matrix multiplication. [2]

7. (8 marks)

(a) The line with equation $y = ax + b$ is mapped to $y = -2x + 5$ after it is transformed by the linear transformation matrix $T = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ T. Determine a and b. [4]

(b) Determine matrix A if $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ [4]